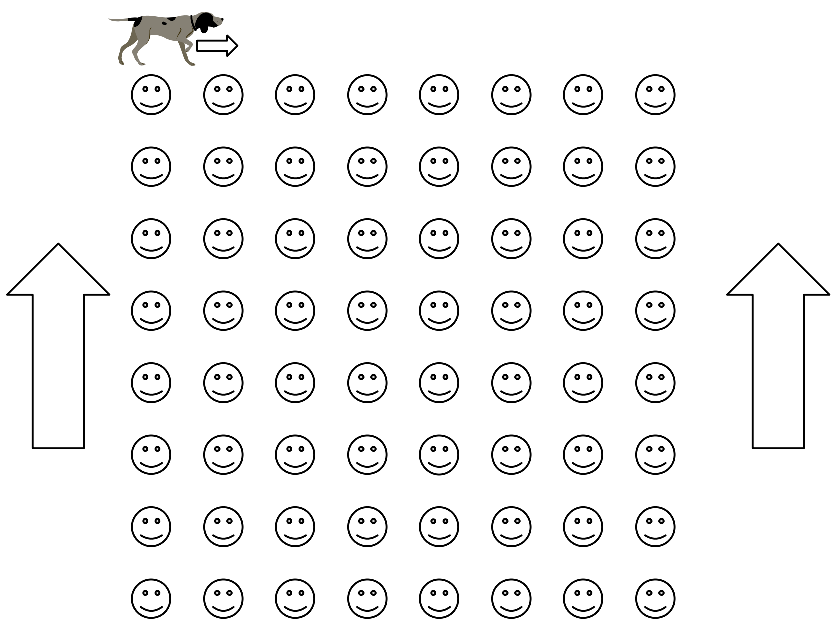
Problem of the Week

September 16, 2009

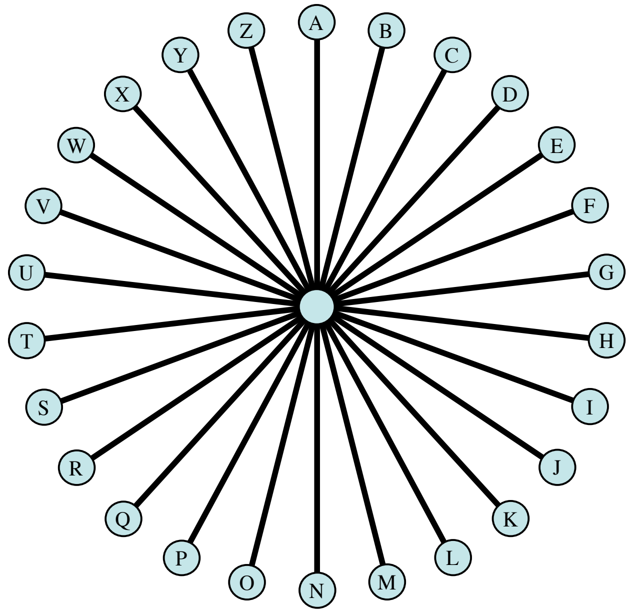
The Running Dog

To celebrate the start of school, our 64 students gather together to form a marching square (ok, our 63 students and a numerically helpful friend). The square is 25 feet on a side and the students in it are marching forward at a constant speed. A friendly dog is running with the children: he starts at the upper left corner and runs around the perimeter of the square, ending up back at the upper left corner. In the time it takes the dog to return to his starting point, the students have moved 25 feet forward. How far has the dog run?



Problem of the Week

September 16, 2009



A simple game. start on the letter A. From there, move according to the following rules:

- If you are on a letter contained in your first name, move 5 units clockwise (So A would move to F)
- If you are on a letter contained in your last name, but not in your first name, move 3 units clockwise. (A moves to D)
- If you are on a letter not contained in either your first or your last name, move 7 units counterclockwise. (A moves to T)

Suppose you move 2009 times….on which letter do you wind up?

Note: to be sure, people with different names are likely to wind up on different letters.

Problem of the Week

September 23, 2009

Mathematical Golf

A mathematician wishes to play a round of golf but, being fundamentally lazy, only wants to learn two strokes; a short one and a long one. There are six holes in this particular course; their lengths are 175, 200, 225, 275, 325, and 425. Which two strokes would give the lowest possible total score?

Suppose the club were to add a seventh hole of 400 yards, what would now be the best two strokes?

To be clear on the rules: the player must use the total length of any stroke but is free to overshoot the hole. Thus, for example, if the two strokes had lengths 25 and 100, the player could complete a 75 yard hole in two strokes (by shooting 100 first and then 25 back). The player would have no way to complete a 90 yard hole. (Why not?)

Problem of the Week

September 30, 2009

Many Tailors

(from Lisa)

6 tailors are able to make 6 shirts in 6 minutes. How many shirts can 100 tailors make in 100 minutes? How many minutes does it take 100 tailors to make 100 shirts? How many tailors are needed to make 100 shirts in 100 minutes?

Note: Depending on how you think about shirt making, it's possible to arrive at slightly different answers. Just be clear what, if anything, you are assuming.

Problem of the Week

September 30, 2009

Miles per Gallon

Units and dimensions are tricky things. Even simple examples can be very, very confusing. Take "miles per gallon" for example. Everyone uses that as a measure of a car's efficiency. A car that can be driven 30 miles on a single gallon of gas is rather efficient. One that can only get 12 miles out of a gallon is not. That's pretty clear. Or is it? We have a simple story we can tell ourselves to justify the measure. From the point of view of the units involved, however, mpg is a fairly bizarre notion.

Question 1. Explain why "miles per gallon" has the same dimensions as 1/Area. Specifically, what is 30 mpg in the units 1/(square feet)? What is 30 mpg in 1/Acres ? (Note: unless you have happened to memorize details such as how many square feet are in an acre you will need to look up various conversion factors.)

Question 2. What does your answer to question 1 mean? After all, we have a clear mental sense of what it means to drive 30 miles on a gallon of gas, but our picture has no apparent connection with 1/Area. Can you come up with a mental picture to link fuel efficiency to this new unit?

Problem of the Week

October 7, 2009

Imbalance

It often happens that errors cancel out, but it can be very hard to see the workings of that. In particular, it can be hard to detect when offsetting factors fail to offset perfectly. For example, in working on the Running Dog problem, a number of people made a false assumption which comes down to the following question:

Suppose a dog were running back and forth along the length of a very slow moving bus, starting and ending at the front of the bus. Does the total distance involved in the dog's path depend on the speed of the bus? Does the time it takes the dog to complete the run depend on that speed? To bring the problem in line with the Running Dog, you may, if you like, assume that the bus ends up with the back end where the front end started. Of course, you are to assume that the dog travels at the same (constant) speed throughout.

As a variant of that question, here is another:

An well-intentioned butcher discovers that his scales are slightly uneven. Seeking to cheat neither his customers nor himself, the butcher weighs a 2 pound order by weighing 1 pound on the left and 1 pound on the right. Does this in fact yield 2 pounds exactly? If not, who is cheated, the butcher or his customers?

Problem of the Week

October 7, 2009

Constraints…

What rule was followed in writing each of the following sentences? Can you write your own? (good luck!)

If critics vilify wursts in Zurich, Swiss risk hissy fit.

Thirty thirsty cyclists swill rum with mint.

Truthfully, Gwynn's nihilistic witticisms inflict hurtful insults.

His chum must visit my clinic in Sicily.

Unskillful flutists hurl shrill music (fifths, sixths, ninths) fussily.

Numskulls, nitwits, schmucks insist misfit is Christ.

Futuristic vinyl miniskirts stultify wrinkly jurists.

Eric intuits mystic list: tin, zinc, lithium, sulfur, yttrium.

Problem of the Week

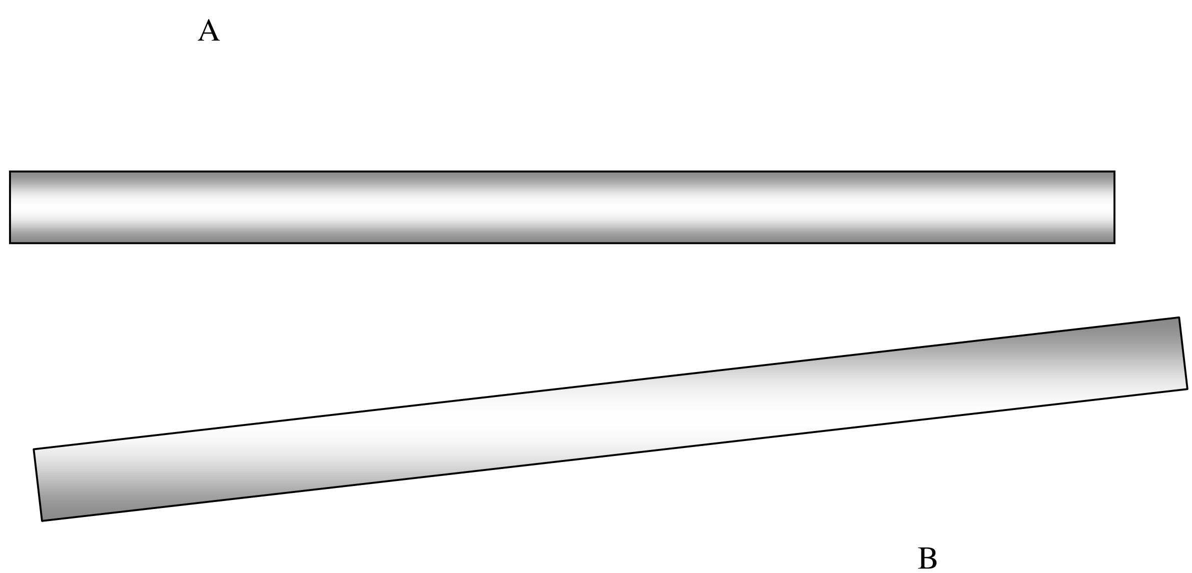
October 14, 2009

Heron's Bridges

This is a very old problem from Greek Geometry.

You would like to build the shortest feasible road between two towns, A and B, but there are two rivers between them. The rivers are not necessarily parallel and, bridge building being very costly, each bridge must be perpendicular to the respective banks. Where should you place the bridges?

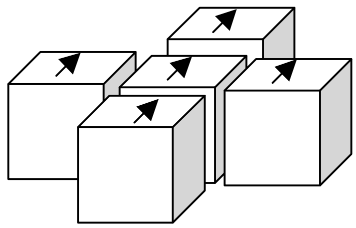
Note: Other than simplicity, there is nothing special about the number two. It should be clear that your method of solution would work for any number of rivers. To start, you might want to work with a single river.



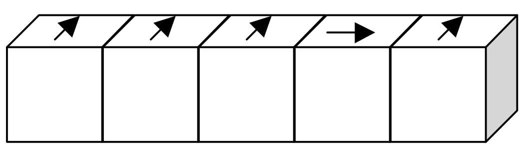
Problem of the Week

October 22, 2009

Heavy Boxes



Five heavy boxes are arranged as you see them. The boxes are identical. They are all cubes and they are unmarked except for an arrow drawn on each of their top faces. The boxes are so heavy that they can't be picked up or even pushed along. The only way to move one of them is to knock it over on a side. You want to get all of them in a line and it is important that you end up with the top side still up (though, of necessity, that side won't be up at every stage). After much hard work you wind up with the following configuration:



As you see, one box has been rotated 90 degrees. Which of these boxes was the center one in the original pattern?

Hint: set this up with cubes and try it! You will need some way to indicate the arrows…

Problem of the Week

November 3, 2009

Many Primes?

Consider the sequence of numbers:

31, 331, 3331, 33331, 333331, 3333331, 33333331,
333333331, …..

It is easy to check that the first few are prime. Are they all prime?

Question 1: Find the first one in the list which isn't prime.

Question 2: Are any of these numbers divisible by 7? 11? 13?

Question 3: (Harder) Are any of these numbers divisible by 23?

Problem of the Week

November 19, 2009

Common Birthdays

Looking over the students this year, I was somewhat surprised to see that we have a triple of common birthdays. That is, there are three students who share a single birthday (month and day, that is, not year).

Should I have been surprised? After all, with 23 randomly chosen people, the odds are about 50:50 that two will share a common birthday…and we have 60 students.

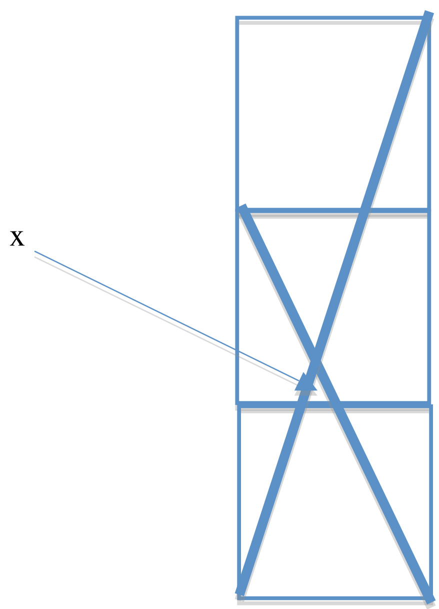
Can you work out the probability that, given 60 students, we will have at least one triple of common birthdays? How many students would we need to get 50:50 odds?

As is usual in such problems, ignore leap years and assume that each of the possible 365 birthdays is equally likely. Handling these points correctly complicates the problem enormously. Even with these simplifying assumptions, it is unlikely that you will be able to solve the problem by hand…some serious calculation is required.

Problem of the Week

December 1, 2009

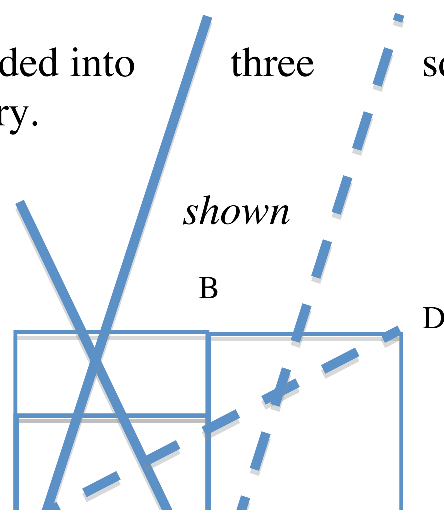
Angle Chasing

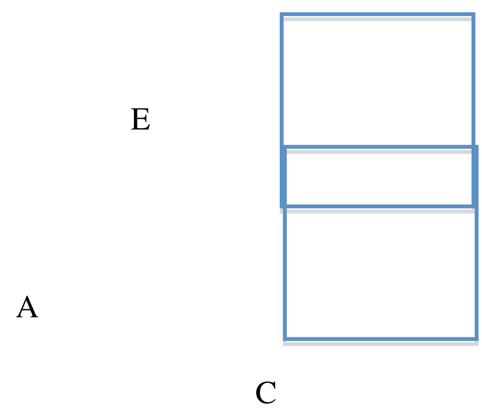


Problem: Find the angle x.

To be clear: the rectangle is divided into three squares.
No hard calculations are necessary.

Solution: Double the figure as shown





Add two new lines, as indicated. The new triangle we have formed is clearly an isosceles right triangle (why?); hence the other two anglesin it (the angles at C and D)are both 45 degrees. It is easy to see that AB is parallel to CD (why?), hence angle <AEC = < ECD = 45, so x is 45.

Note: this is a straight forward trigonometry exercise. It is clear that Tan(<EAC) = 3 and that Tan(<ECA) = 2. We can now use the addition formulas to computeTan(<AEC). As frequently happens, the trigonometric calculation is easy enough but the answer just appears to drop out without any geometric meaning.

Problem of the Week

January 3, 2010

Risk/Reward

A gambler has $2 but requires $4. He finds someone who will bet him double or nothing on any amount he chooses, as often as he likes. He considers two strategies: The Aggressive One, in which he wagers the whole $2 at once and the Cautious One, in which he only wagers $1 at a time until he either has $4 or has been wiped out.

Question 1: Supposing that his odds of winning any given play are exactly _, which strategy gives him the greatest probability of success? (of course, they might be equal).

Question 2: Supposing, as is more realistic, that his odds of winning any given round are slightly less than _; just to be precise, let's say the odds are .49 . Now which strategy should he prefer?

Notes:

1. There is nothing special about the $2 or the $4…having small numbers simplifies the algebra, though the conclusions stay the same.
2. You can try this game yourself and determine the answer experimentally. To do it, it is probably best to exaggerate the unfairness of the play in Question 2. Suppose your odds of winning a toss are 2/5, say.
3. To put the odds in perspective; in standard roulette the odds of winning on, say, Red are 18/38 ~ .4737

Problem of the Week

January 19, 2010

Socks

A certain fellow's sock drawer contains red and blue socks only. The fellow plans to draw 2 socks at random from them…curiously, if he does so, the odds are exactly _ that he will get two red socks. What is the smallest number of socks that makes this possible?

Harder: What is the next smallest possibility?

(Note: the third smallest solution is pretty hard to find, though not entirely out of the question).

Solution: Let r be the number of red socks, and b the number of blue socks. It follows that there are r + b socks altogether.

The odds that the fellow will draw two red ones are then:

$$\frac{r}{r+b}\frac{r-1}{r+b-1}=\frac{1}{2} \quad \Rightarrow \quad 2r(r-1)=(r+b)(r+b-1)=x(x-1)$$

where x is r + b = the total number of socks. We can now make a table of the values of n(n-1) as n runs from 1, 2, 3, … until we find two entries one of which is twice the other. We get:

n	n(n-1)
2	2
3	6
4	12
5	20
6	30
7	42
8	56
9	72
10	90
11	110
12	132
13	156
14	182
15	210
16	240
17	272
18	306
19	342
20	380
21	420

*That's all it takes! As 12 = 2*6 we see that we can just take x = 4 and r = 3. This means that b = 1, so there are 4 socks all in all. The next one up has x = 21 and r = 15. Hence, 15 red socks and 6 blue ones.*

Problem of the Week

February 9, 2010

The Logical Jailer

You are being held prisoner by the sort of jailer who only comes up in logic problems. This particular mastermind is obsessed with games of Chance and has devised a diabolical game which you must beat to earn your freedom.

Here is the game. You are given a coin and you are told that there is a second prisoner just like you who also has a coin and who is in the same position you are. The game is to be played in an hour. You (and the other prisoner) have a choice. You can flip your coin or not. You both will be released if at least one Heads is tossed and no Tails are tossed. That's it.

To be clear: You can Pass or you can flip, in which case you will get a Heads or a Tails (with equal odds). Thus, you have three possibilities; Pass, Heads, Tails. The other prisoner has the same three possibilities. Between you there are therefore nine conceivable outcomes. You are both released if the outcomes are (Pass, Heads), (Heads, Pass), or (Heads, Heads). You lose in all of the other six outcomes.

You have the hour to think, but you can not communicate with the other prisoner in any way. Of course, you may assume that the other prisoner is every bit as good at logic as are you.

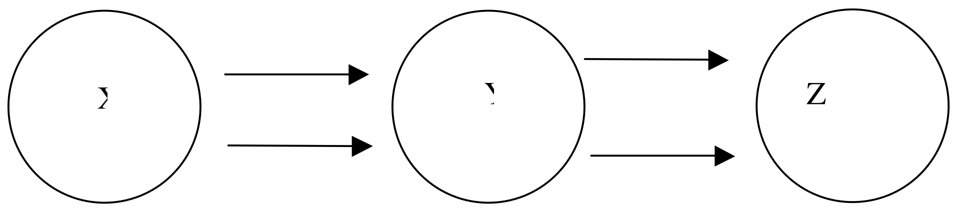
What should you do and what are the odds of escape?

Hint: you can do significantly better than 25%.

Problem of the Week

March 17, 2010

The Aftermath of the Storm



A problem in honor of the recent near-hurricane:

Let's suppose there are three places, cleverly named X, Y, and Z. There are exactly two roads connecting X to Y, and exactly two roads connecting Y to Z (and there are no other roads). The storm has thrown trees and branches everywhere, and there is a good chance that any (or all) of these roads might be closed. To keep things simple, let's suppose that each road has a 1/3 chance of being closed and that all the closings are "independent" (so that knowing whether one particular road is closed or not tells you nothing about any of the other roads).

Question 0: What are the odds that all the roads are closed? What are the odds that they are all open?

Question 1: What are the odds that there is an open route from X to Z?

Question 2: (Harder) You are located at place X and wish to drive to Y. You hear on the radio that there is no open route from X to Z, but that's all the information you have. What are the odds that at least one of the roads from X to Y is open?

Problem of the Week

April 7, 2010

Conflict Resolution

To resolve matters of honor, our students have evolved an unusual method of dueling which requires the two participants to meet and exchange rude epithets in antique languages. Going yet further, the students have decided that simply showing up is enough to defend one's honor; the duel itself needn't take place.

Thus: Each of the two participants simply shows up on the dueling field at some random time between 7:00 and 8:00 in the morning. The new arrival waits for exactly one minute (or until 8:00, whichever comes first). If their opponent does not happen to appear in that minute, then honor is served and the actual duel is avoided (it is not rescheduled). If, by chance, both find themselves on the field at the same time, then the battle goes forward.

What are the odds that the duel takes place?

Problem of the Week

April 29, 2010

Words

(from a CarTalk puzzle)

What do the following words have in common?

cab hag jade leaf leg maced mica mid need pig real ride same sand seam sod table toad toe vial ward whack who wick win yeas yet zebra zinc

Just to keep matters clear: the original puzzle only revealed a few of these, along with the extra hint that no eligible word began with the letter "a". This list contains every eligible word, or at least every eligible word that my brother was able to find via a computer search of a complete dictionary. The order of the words is irrelevant.

Problem of the Week

May 11, 2010

Subway Fare

A ride on a New York City subway costs \$2.25 .   You can pay for this ride by a MetroCard and, if you do it that way, you'll actually get a bonus! Specifically, the city will give you an extra 15%. Thus, if you buy a \$9 MetroCard, you will get \$10.35 worth of subway fares. Notice, however, that while \$9 would correspond to exactly 4 fares, \$10.35 does not give you an even number of fares. Accordingly, we have the following:

Question: What is the smallest dollar amount you can pay for a MetroCard that will give you an integral number of fares?

To be clear: we are looking for a practical answer here. You could theoretically pay \$2.25/1.15 = \$1.95652… to get a single fare, but we are looking for an amount that can be expressed exactly using dollars and cents.

Full Disclosure: This problem was sent in by an occasional reader of our problems (a reader who, one imagines, sometimes finds himself vexed by the NYC subway system)

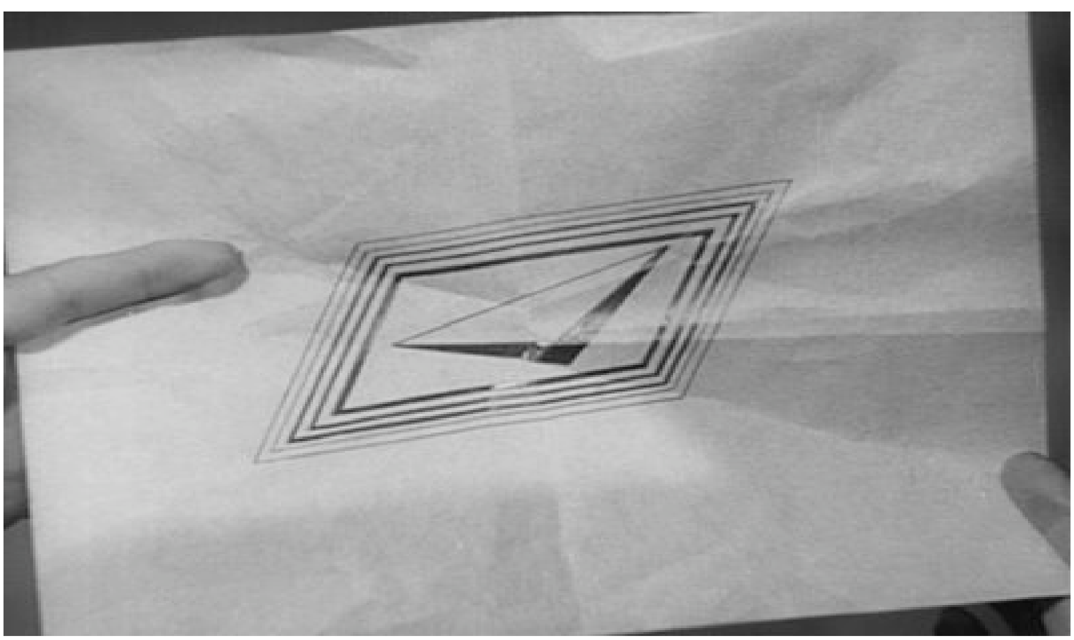
Problem of the Week

May 19, 2010

Sinister Logo

The image below is a frame from *The Seventh Victim* (1943). An excellent film; probably the first to assert a connection between the forces of evil and the cosmetics industry. The shape is the trademark of La Sagesse…a business owned and operated by the aforementioned evil doers. In the movie, one of the characters describes the figure as "A parallelogram with a split triangle in its very center." Can you explain the rules by which the figure is drawn?

Note: I believe I have the pattern, but I can't be certain. I don't see anything especially evil about it, but perhaps that's just me.



Problem of the Week

May 26, 2010

Word Circles

It is hard to define things. Well, maybe not concrete things like "pencils" or "shoes", but abstract things, like "time" and "space".

Case in point: In reading possible Science books for next year, I came across the following definition of Matter.

Matter *is anything that takes up space and has mass. Mass is the amount of matter in an object.*

I thought this might be a problem with this particular book, so I checked the OED. Quoting, in part, we start with:

Matter: *the substance, or the substances collectively, out of which a physical substance is made or of which it consists***.**

Well, ok…but what is a "physical" substance? Is there another kind of substance? Let's look up "physical":

Physical: *pertaining to, or connected with matter***.**

You see the problem. One abstract word leads to another and then another until you get back where you started.

Problem: both of these are very short circles (word A leads to word B which leads back to word A). Can you find longer ones? Specifically, using the OED, find a circle which is at least 4 or 5 words long. How long a loop can you find?